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Natural convection heat transfer from complex surface

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Abstract—The analytical solution of convective heat transfer from an isothermal complex surface in an unlimited space has been presented. The complex surface is represented by a horizontal ring of diameters D and d with hemispherical segment of diameter d in the centre. The shape of the complex surface was expressed by factor $\rho = d/D$. The presented solution has been verified experimentally on a set-up of diameter of 0.4 m and height of 0.5 m with surfaces of constant external diameter $D = 0.06$ m and various shape factor $\rho = 0$—round plate, 0.183, 0.233, 0.40, 0.483, 0.554, 0.650, 0.817 and 1.0—hemisphere. The tested fluid was glycerine. The comparison of theoretical and experimental results gives good agreement.

INTRODUCTION

A complex surface can be composed, for example: of vertical cylinder covered by hemisphere, vertical cylinder ended from the top and from the bottom by two hemispheres, vertical cylinder ended by two cones, round horizontal plate with hemispherical segment in the centre and so on (Fig. 1). These surfaces are often the heating surface in electronic devices (transistors, diodes, integrated circuits), building engineering (domes, cupolas, bowls), chemical engineering (tanks, containers, reactors, autoclaves), lighting industry (lanterns, beacons, lamps) or in meteorology. However, a search of the literature revealed scarce information on convective heat transfer from such complex surfaces.

The research presented was aimed at making an attempt to apply a model of convective heat transfer in an unlimited space from an isothermal complex surface represented for example by horizontal ring with hemisphere in the middle. This choice was caused by the fact that the components of such a complex surfaces; horizontal ring and hemisphere have been studied and obtained results have been published recently [1, 2].

The considered surface was described by the shape factor $\rho = d/D$ being a ratio of the sphere diameter d and the outer diameter of the ring D.

MODEL OF CONVECTION FROM THE COMPLEX SURFACE

The analysis of results of the visualisation studies gives an evidence that a boundary layer forms at a round leading edge of the ring and flows concentric to the round edge of the hemisphere. The fluid flows then above the hemisphere and next the boundary layer transforms into buoyant plume over the centre point of the complex surface. This physical model is presented in Fig. 2.

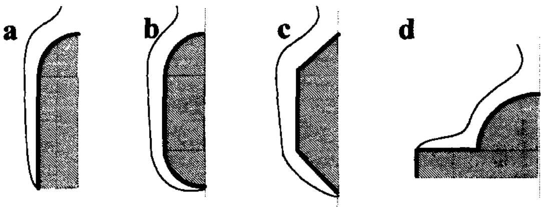

Fig. 1. Some examples of complex surfaces: (a) vertical cylinder–hemisphere, (b) hemisphere–vertical cylinder–hemisphere, (c) cone–vertical cylinder–cone, (d) horizontal ring–hemisphere.

The heat flux from the whole surface Q_c is composed of heat fluxes from the ring Q_r and from the hemisphere Q_h:

$$Q_c = Q_r + Q_h = \alpha_r A_r \Delta T_r + \alpha_h A_h \Delta T_h = \alpha_c A_c \Delta T_c. \quad (1)$$

For an isothermal surface ($\Delta T_c = \Delta T_r = \Delta T_h$) and for $A_c = \pi(D^2+d^2)/4$, $A_r = \pi(D^2-d^2)/4$, $A_h = \pi d^2/2$ one has:

$$\alpha_c = \alpha_r \frac{(D^2-d^2)}{(D^2+d^2)} + 2\alpha_h \frac{d^2}{(D^2+d^2)} \quad (2)$$

or

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NOMENCLATURE

$a = \gamma/(c_p\rho)$ thermal diffusivity
A area
c_p specific heat at constant pressure of the fluid
$C = Nu/Ra^{1/4}$ coefficient in Nusselt–Rayleigh relation equations (9), (10)
d internal diameter of ring in complex surface
D external diameter of ring and complex surface
f function defined by equation (11)
g gravitational acceleration
h thickness of the slab in heat flux measuring plate
I current of electric power of heater
$Nu = \alpha D/\lambda$ Nusselt number
Q heat flux
$Ra = g\beta\Delta TD^3/(\nu a)$ Rayleigh number
T temperature
U voltage of electric power of heater
x coefficient of heat losses from heater equation (17).

Greek letters
α heat transfer coefficient
β coefficient of volumetric expansion
γ angular downstream location
δ thickness of boundary layer
λ thermal conductivity
ν kinematic viscosity
ρ shape factor of surface.
Φ function defined by equations (7), (8).

Subscripts
c complex surface
h hemispherical segment
r ring segment of surface
0 end for ring and first for hemisphere
loss losses.

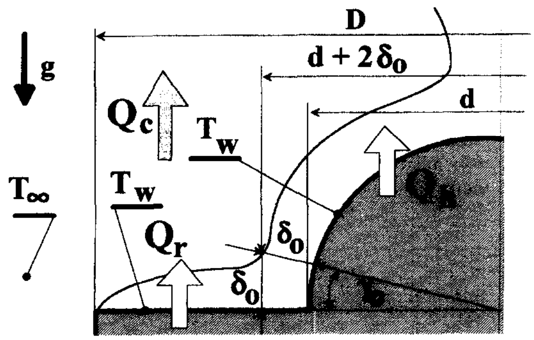

Fig. 2. Physical model of convective heat transfer from complex surface represented by horizontal ring with hemisphere in the middle.

$$Nu_c = \frac{(1-\rho^2)}{(1+\rho^2)}Nu_r + \frac{\rho^2}{(1+\rho^2)}Nu_h \quad (3)$$

where: $Nu_c = \alpha_c D/\lambda$, $Nu_r = \alpha_r D/\lambda$, $Nu_h = \alpha_h D/\lambda$ and $\rho = d/D$.

The Nusselt numbers in natural convection heat transfer from a horizontal ring and a hemisphere are described by Nusselt–Rayleigh relations and these relations are known from previous studies [3] and [4]:

$$Nu_r = C_r \cdot Ra_r^{1/5} \quad \text{(for the ring)} \quad (4)$$

$$Nu_h = C_h \cdot Ra_h^{1/4} \quad \text{(for the hemisphere).} \quad (5)$$

The substitution of Nusselt–Rayleigh relations equations (4) and (5) into equation (3) in which the same characteristic linear dimension D is used, leads to relation for isothermal complex surface:

$$Nu_c = (\Phi_r + \Phi_h)Ra^{1/4} = \Phi_c Ra^{1/4} \quad (6)$$

where:

$$\Phi_r = \frac{(1-\rho^2)C_r}{(1+\rho^2)\cdot Ra^{1/20}} \quad (7)$$

$$\Phi_h = \frac{2\rho^2 C_h}{(1+\rho^2)}. \quad (8)$$

The constants for the ring C_r estimated in paper [3] is expressed by relation:

$$C_r = \frac{1.151\cdot(1-\rho^{7/3})^{3/4}}{(1-\rho^2)^{6/5}}\left[(1-\rho^{7/3})^{1/4}\rho^{2/3} + \frac{3}{2}(1-\rho^{2/3}) - \frac{1}{12}(1-\rho^3) - \frac{9}{512}(1-\rho^{16/3}) \pm \frac{21}{2944}(1-\rho^{23/3}) - \frac{77}{20480}(1-\rho^{10})\right]^{1/5}. \quad (9)$$

For the boundary case of the ring or of the complex surface i.e., round horizontal plate ($\rho = d/D = 0$) the constant equation (9) in Nusselt–Rayleigh relation equation (4) is $C_r = 1.229$.

The second constant in relation equation (5), being connected with the spherical part of complex surface, can be calculated according to procedure described in paper [4] from relation:

$$C_h = \frac{0.5}{(1-\sin\gamma_0)} \int_{\gamma_0}^{\pi/2} \frac{\cos\gamma \cdot d\gamma}{f(\gamma, \gamma_0, \delta_0)} \tag{10}$$

where:

$f(\gamma, \gamma_0, \delta_0)$

$$= \frac{\left[\int_{\gamma_0}^{\pi/2} \cos^{5/3}\cdot\gamma\cdot d\gamma + \left(\frac{2\cdot\delta_0}{d}\right)^4 \frac{Ra\cos^{8/3}\gamma_0}{2560}\right]^{1/4}}{\cos^{2/3}\gamma}. \tag{11}$$

Equations (10) and (11) differ from the original solution for hemisphere [4] by assumption that coefficient of boundary layer shape ($\partial\delta/\partial x \to 0$) can be neglected for complex surface. The convective fluid flow on separate hemisphere and hemisphere being the part of complex surface differs from each other. In the first case convective boundary layer thickness starts from $\delta = 0$ at $\gamma = 0$. In the second case (complex surface—see Fig. 2), the boundary layer thickness starts on the hemispherical surface from the value $\delta = \delta_0$ at $\gamma = \gamma_0$. The consequence of this is that the integration constant in original solution [4] being $C_1 = 0$ has now the form:

$$C_1 = \left(\frac{2\cdot\delta_0}{d}\right)^4 \frac{Ra\cdot\cos^{8/3}\gamma_0}{64\cdot 40^{4/3}}. \tag{12}$$

The value of angle γ_0, according to Fig. 2, can be expressed as:

$$\gamma_0 = \operatorname{arc\,tg}[\delta_0/(d/2+\delta_0)]. \tag{13}$$

The final value of the boundary layer thickness on the ring (δ_0) is, according to the law of continuity, the same as the initial value of the boundary layer thickness on the hemispherical part of the complex surface (Fig. 2). The value of the boundary layer thickness above the diameter of $d+2\delta_0$, calculated according to the procedure given in [3] is expressed by relation:

$$\frac{2\cdot\delta_0}{d} = \frac{3.971(1-\rho^{7/3})^{1/4}}{Ra^{1/5}\rho^{1/3}}(1-\rho^2)^{1/5}\Big[(1-\rho^{7/3})^{1/4}\rho^{2/3} + \frac{3}{2}(1-\rho^{2/3}) - \frac{1}{12}(1-\rho^3) - \frac{9}{512}(1-\rho^{16/3}) - \frac{21}{2944}(1-\rho^{23/3}) - \frac{77}{20480}(1-\rho^{10})\Big]^{-1/5}. \tag{14}$$

By the substitution of relation (14) into equation (13) one can calculate the angle γ_0 which is necessary to calculate the constant C_h from equations (10) and (11).

The results of calculations of coefficients Φ_r, Φ_h and Φ_c for complex surface of chosen dimensions expressed by the $\rho = d/D$ ratio and for given values of Rayleigh numbers ($Ra = 10^3$, 10^4, 10^5, 10^6, 10^7 and 10^8) are compared in Table 1 and are illustrated in Fig. 3 and Fig. 4.

Introducing the value of the coefficient for the ring

Table 1. The values of coefficients Φ_r, Φ_h and Φ_c as a function of the shape factor of the complex surface $\rho = d/D$ and Rayleigh number Ra

	ρ										
Ra	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
						Φ_r					
10^3	0.870	0.846	0.808	0.749	0.675	0.591	0.498	0.401	0.299	0.186	0
10^4	0.775	0.754	0.719	0.667	0.602	0.526	0.444	0.357	0.266	0.166	0
10^5	0.691	0.672	0.641	0.595	0.536	0.469	0.396	0.319	0.237	0.148	0
10^6	0.616	0.599	0.571	0.530	0.478	0.418	0.353	0.284	0.211	0.132	0
10^7	0.549	0.534	0.509	0.473	0.426	0.372	0.314	0.253	0.188	0.118	0
10^8	0.489	0.476	0.454	0.421	0.380	0.332	0.280	0.225	0.168	0.105	0
						Φ_h					
10^3	0	0.006	0.028	0.069	0.126	0.197	0.278	0.363	0.445	0.516	0.522
10^4	0	0.005	0.025	0.062	0.115	0.182	0.258	0.341	0.424	0.497	0.522
10^5	0	0.005	0.023	0.056	0.104	0.166	0.238	0.319	0.403	0.481	0.522
10^6	0	0.004	0.020	0.050	0.094	0.151	0.219	0.297	0.383	0.468	0.522
10^7	0	0.004	0.018	0.045	0.085	0.137	0.200	0.275	0.362	0.455	0.522
10^8	0	0.003	0.016	0.041	0.076	0.123	0.182	0.253	0.340	0.441	0.522
						Φ_c					
10^3	0.870	0.852	0.835	0.818	0.802	0.788	0.776	0.764	0.744	0.702	0.522
10^4	0.775	0.759	0.745	0.730	0.717	0.708	0.703	0.699	0.691	0.663	0.522
10^5	0.691	0.677	0.664	0.651	0.641	0.635	0.634	0.638	0.641	0.630	0.522
10^6	0.616	0.603	0.592	0.581	0.573	0.569	0.572	0.581	0.594	0.600	0.522
10^7	0.549	0.538	0.527	0.518	0.511	0.509	0.515	0.528	0.551	0.572	0.522
10^8	0.489	0.479	0.470	0.462	0.456	0.455	0.462	0.479	0.508	0.545	0.522

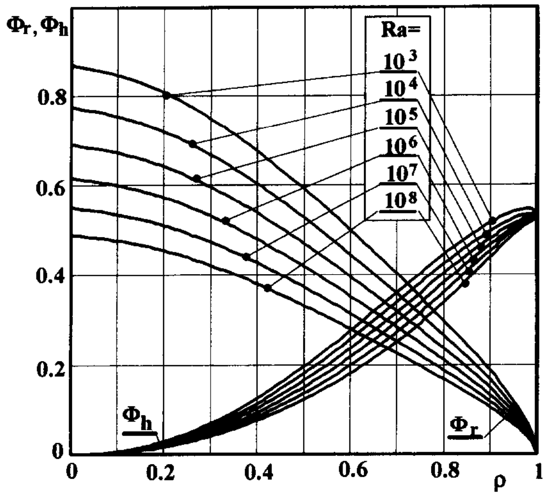

Fig. 3. Dependence of Φ_r and Φ_h on ρ and Ra for the horizontal ring with hemisphere in the middle.

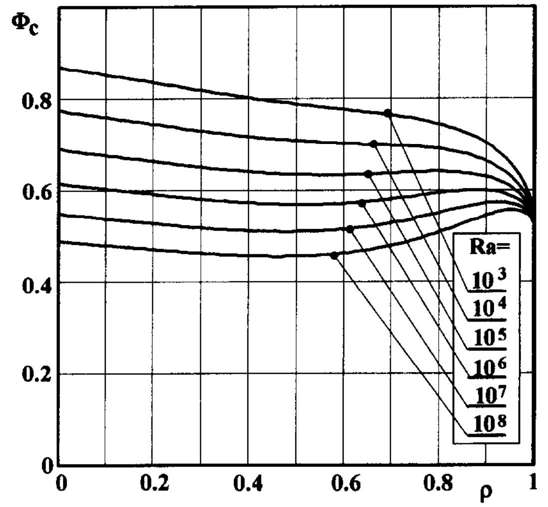

Fig. 4. Dependence of Φ_c on ρ and Ra for the horizontal ring with hemisphere in the middle.

surface C_r and the value of the coefficient connected with the spherical segment of the complex surface C_h into equation (4), one can obtain the solution in the form of correlation of coefficient C_c as a function of Rayleigh number Ra and the shape factor of the complex surface ρ.

EXPERIMENTAL APPARATUS AND PROCEDURE

Experiments were made in distilled glycerol and water for the complex surfaces of constant external diameter of $D = 0.06$ m and of different shape factor $\rho = 0.0$ (plate), 0.183, 0.223, 0.40, 0.483, 0.554, 0.650, 0.817 and 1.00 (hemisphere). The complex surface was made from two copper sheets of the thickness 1 mm each by pressing. Experimentally the thickness 2 mm of complex surface allowed obtained in experimental procedure isothermal temperature of the wall T_w was found. To measure this temperature three thermocouples were mounted between sheets of each tested complex surface.

Experimental stand and procedure are similar to that used to measure convective heat transfer from a horizontal ring and hemisphere and are described in detail in works [3] and [4]. Schematic cross-section of the (testing equipment) apparatus used to carry out experimental investigations is shown in Fig. 5. The apparatus used in the experiment was a Plexiglas tank with a copper coil connected to a thermostat. The main dimensions of the tank were 0.4 m in diameter and 0.5 m in height. In the centre of the bottom, on a round horizontal heated plate, a glass tube held the complex surface (Fig. 5(c)). The tube had double walls (Fig. 5(a) and 5(b)) with vacuum between them to minimise heat losses Q_{loss}.

The heat flux ($Q_{inl} = U \cdot I$) from the heating plate inside the glass tube, filled by the test fluid, was transported by turbulent convection into copper complex surfaces. Then the heat flux from the investigated complex surfaces ($Q_{out} = U \cdot I - Q_{loss}$) to the surrounding test fluid was transferred also by convection, but in this case by a laminar one. A slab plate used for measuring, by direct method, the heat flux Q for

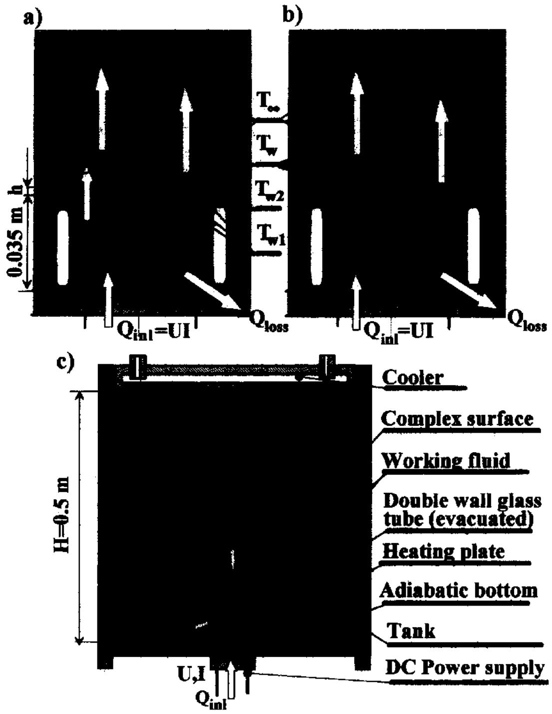

Fig. 5. Schematic diagram of experimental equipment (a) the slab plate for measuring by independent (indirect) method the heat flux for boundary cases of complex surface (round horizontal plate and hemisphere), (b) position of complex surfaces with the use of dependent (direct) method of heat stream measuring, (c) cross-section of the apparatus used.

boundary cases of complex surface $\rho = 1$ and $\rho = 0$ and to estimate the heat loss flux Q_{loss} is illustrated in Fig. 5(a). Figure 5(b) with enlarged fragment of tested surfaces shows the indirect method of the heat flux $Q = Q_{inl} \cdot x_{loss}$ estimation. The heat flux from complex surface to the tested fluid was measured with the use of two methods for the boundary cases only. These methods and obtained results were described [3] for round horizontal plate and [4] for the hemisphere investigations. In this paper the results obtained for all cases ($0 \leqslant \rho \leqslant 1$) but with the use the only indirect method were presented. In this method the heat flux was calculated from relation:

$$Q = U \cdot I \cdot x_{loss} \tag{15}$$

where: U and I are voltage and current of heater and x_{loss} is the coefficient of global heat losses from the heater through the glass tube, bottom and support construction.

The coefficient of heat losses was estimated with the use of the measuring slab plate according to formula:

$$Q_{loss} = U \cdot I - (\lambda/h) \cdot (\Delta T_s) \cdot (\pi d^2/4) \tag{16}$$

$$x_{loss} = (U \cdot I - Q_{loss})/(U \cdot I) \tag{17}$$

where: λ is the conductivity at the mean temperature $(T_{w1} + T_{w2})/2$ of the fluid inside the slab of measuring plate, h is the slab thickness and $\Delta T_s = T_{w1} - T_{w2}$ is the difference of the temperature of both copper plates of measuring slab plate (Fig. 5(a)).

The convective part of the heat flux transferred through the slab could be neglected in our experiments because the distance between two copper round plates of the measuring plate calculated from equation (18) was $h = 1.9$ mm. For this value of the slab thickness and for all experimental runs the Rayleigh number values inside the slab were smaller than the value of critical Rayleigh number Ra_{cr} and this means that the whole heat flux is transferred by conduction only.

$$g \cdot \beta \cdot (T_{w1} - T_{w2}) \cdot h^3/(a \cdot \nu) < Ra_{cr} = 1700. \tag{18}$$

The empirical correlation of calibration results (Fig. 6), obtained for the two characteristic cases of the complex surface ($\rho = 0$—round plate) and $\rho = 1$—hemisphere) have the forms

$$x_{loss} = 0.02248 \cdot Ra^{0.1679} \quad \text{for } \rho = 0 \quad \text{(round plate)} \tag{19}$$

$$x_{loss} = 0.02393 \cdot Ra^{0.1473} \quad \text{for } \rho = 1 \quad \text{(hemisphere).} \tag{20}$$

The general relation of calibration reads:

$$x_{loss} = (0.00045 \cdot \rho + 0.02248) \cdot Ra^{(0.1679 - 0.0206\rho)} \quad \text{for } 0 \geqslant \rho \geqslant 1. \tag{21}$$

For each complex surface described by shape factor ρ and for the experimental values of Rayleigh number the coefficient of heat losses x_{loss} were estimated from equation (21). From the experimental values current and voltage of heating plate the heat fluxes Q were calculated from equation (15).

From the difference of the temperature of complex surface and of the fluid in an undisturbed region the heat transfer coefficients $\alpha = Q/(A_c \cdot (T_w - T_\infty))$ and the next the Nusselt numbers $Nu = \alpha \cdot D/\lambda$ were calculated.

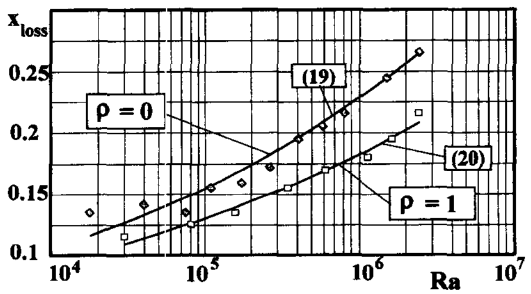

Fig. 6. The results of calibration of experimental stand up reduced to found relations between heat loss coefficients x_{loss} for a characteristic case of complex surface (round plate and hemisphere) and Rayleigh numbers.

EXPERIMENTAL RESULTS

By using the least square method, the experimental points obtained for the tested surfaces with various shape factors ρ have been correlated by Nusselt–Rayleigh relations for given values of coefficient $C = Nu/Ra^n$ and exponent n or for the constant value of exponent $n = 1/4$ or $n = 1/5$.

The approximation of experimental results and mean square deviations δ^2 have been presented below:

$\rho = 0$ (round plate)

$$Nu = 1.540Ra^{0.174} \; (\delta^2 = 0.997),$$
$$Nu = 0.588Ra^{1/4} \; (\delta^2 = 0.808)$$
$$\text{or} \quad Nu = 1.109Ra^{1/5} \; (\delta^2 = 0.975)$$

$\rho = 0.183$

$$Nu = 2.263Ra^{0.156} \; (\delta^2 = 0.973),$$
$$Nu = 0.707Ra^{1/4} \; (\delta^2 = 0.626)$$
$$\text{or} \quad Nu = 1.318Ra^{1/5} \; (\delta^2 = 0.898)$$

$\rho = 0.233$

$$Nu = 1.290Ra^{0.185} \; (\delta^2 = 0.974),$$
$$Nu = 0.560Ra^{1/4} \; (\delta^2 = 0.852)$$
$$\text{or} \quad Nu = 1.060Ra^{1/5} \; (\delta^2 = 0.967)$$

$\rho = 0.4$

$$Nu = 1.224Ra^{0.186} \; (\delta^2 = 0.986),$$
$$Nu = 0.538Ra^{1/4} \; (\delta^2 = 0.870)$$
$$\text{or} \quad Nu = 1.024Ra^{1/5} \; (\delta^2 = 0.980)$$

$\rho = 0.483$

$$Nu = 1.096Ra^{0.199} \; (\delta^2 = 0.999),$$
$$Nu = 0.578Ra^{1/4} \; (\delta^2 = 0.933)$$
$$\text{or} \quad Nu = 1.080Ra^{1/5} \; (\delta^2 = 0.999)$$

$\rho = 0.55$

$$Nu = 0.732Ra^{0.228}\ (\delta^2 = 0.995),$$
$$Nu = 0.554Ra^{1/4}\ (\delta^2 = 0.986)$$
$$\text{or}\quad Nu = 1.038Ra^{1/5}\ (\delta^2 = 0.980)$$

$\rho = 0.65$

$$Nu = 0.682Ra^{0.228}\ (\delta^2 = 0.999),$$
$$Nu = 0.520Ra^{1/4}\ (\delta^2 = 0.990)$$
$$\text{or}\quad Nu = 0.970Ra^{1/5}\ (\delta^2 = 0.984)$$

$\rho = 0.817$

$$Nu = 0.581Ra^{0.282}\ (\delta^2 = 0.977),$$
$$Nu = 0.840Ra^{1/4}\ (\delta^2 = 0.984)$$

$\rho = 1$ (hemisphere)

$$Nu = 0.443Ra^{0.258}\ (\delta^2 = 0.997),$$
$$Nu = 0.490Ra^{1/4}\ (\delta^2 = 0.996)$$

The experimental results presented in the form of average Nusselt numbers Nu vs. Rayleigh number Ra are shown in Fig. 7. The plot contains, as an example, the results obtained for glycerine only. The solid lines in Fig. 7 represent the theoretical solution equation (6).

Some of the experimental points in Fig. 7 are pointed by letters. For these points in Fig. 9 the visualisation results are also presented.

In Fig. 8 the experimental results plotted in the form of average values of the coefficient $C = Nu/Ra^{1/4}$ vs ρ are compared with the analytical solution equation (6). The direct comparison is not possible because the analytical solution, obtained in the form of $\Phi_c = Nu/Ra^{1/4}$, as one can see in Fig. 4 and Table 1 is a function of Rayleigh number. So the solid line in Fig. 8 represents analytical solution averaged for the range of performed experiments ($10^5 \geqslant Ra \geqslant 10^7$). The experimental results, presented above, were calculated with the use of different values of exponents n varying from $n = 0.156$ to $n \doteq 0.282$ but for comparison with theoretical solution these results were approximated with the use of exponent $n = 1/4$ (as in theoretical solution).

In Fig. 9 some results of visual studies of convective heat transfer have been presented. The analysis of stream line patterns was very useful for preparing the model of free convection phenomenon from the complex surfaces and for the explanation of deviations from the rules of proposed mechanism ($\rho = 0.183$ and $\rho = 0.817$).

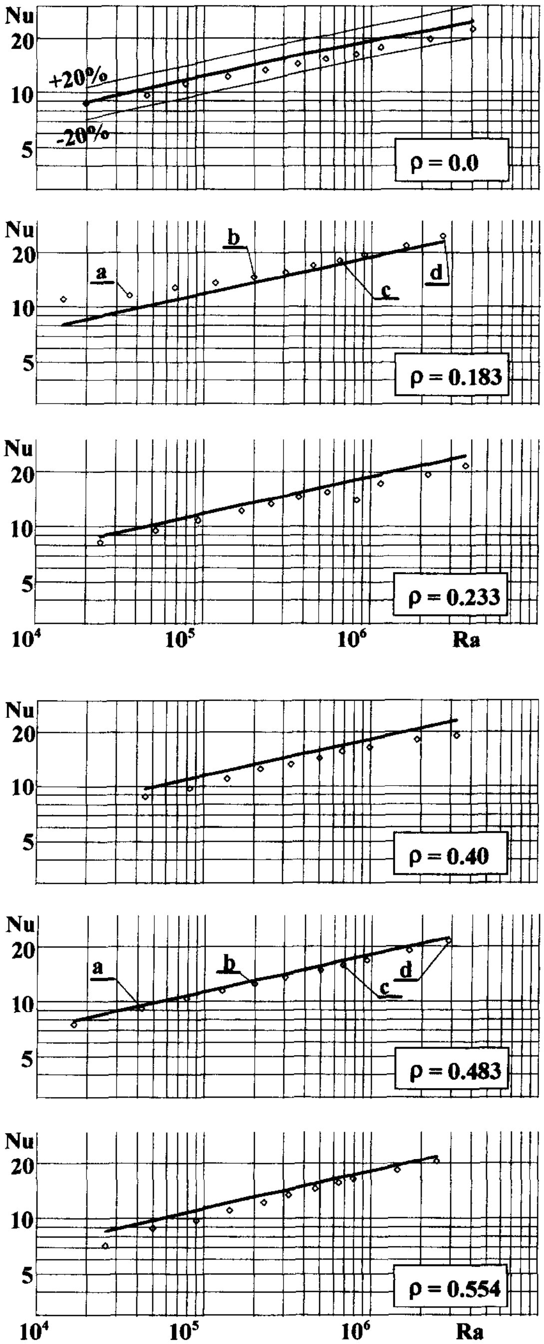

Fig. 7. Experimental data (points) compared with theoretical results (solid lines) obtained for some complex surfaces and for glycerine. The letters describe some experimental points corresponding to the results of visual studies presented in Fig. 9.

DISCUSSION OF THE OBTAINED RESULTS

From the analysis of Fig. 7 and Fig. 8 it is obvious that the experimental results for the most of the complex surfaces tested, agree well with the proposed theoretical model of natural convection heat transfer from horizontal ring with hemisphere in the middle as an example of complex surface and with their analytical solutions. Two cases of investigated surfaces, namely for the shape factors $\rho = 0.183$ and $\rho = 0.817$, were the exceptions to this rule. Repeated experiments for these surfaces also in water (not presented in this paper) [5] eliminate gross error and indicate that the mechanism of this phenomenon in the proposed model is disturbed by additional effects.

The first singularity of intensification of heat trans-

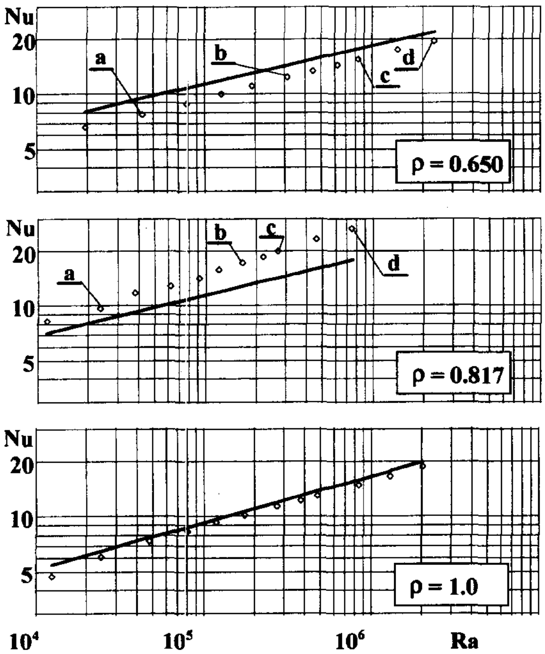

Fig. 7—*continued.*

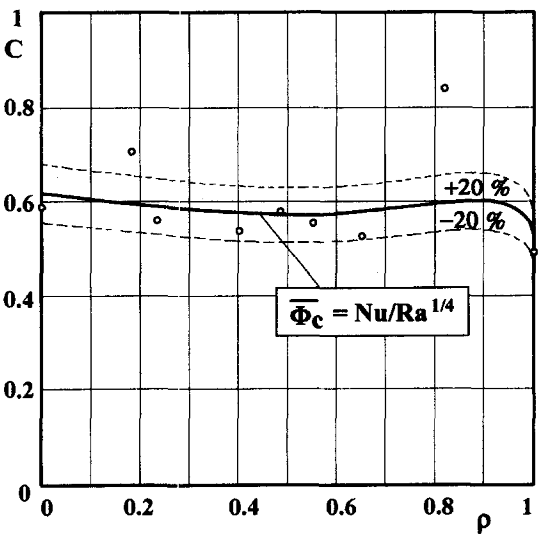

Fig. 8. Comparison of the averaging, for the range of performed experiment ($10^5 \geqslant Ra \geqslant 10^7$), analytical solution (solid line) with the experiments described by $C = Nu/Ra^{1/4}$ (points).

fer for $\rho = 0.183$ may be caused by the fact that the exponent $n = 1/4$ in this cases is not as good for approximation of experimental points as $n = 1/5$ or is accompanied by following possible effects for example:

—local overheating of the fluid at surroundings of small diameter convexity of the flat horizontal surface—similar phenomenon was observed by the authors for convective heat transfer in the closed space [6];

—elimination of the 'dead' space with motionless fluid in the middle of the plate by convexity. In the case in which this conical dead space exists, the heat is transferred from the plate through this insulating fluid layer by conduction only. This effect is described by the authors in the work [7];

—the small convexity may indicate a phenomenon similar to condensation, boiling or crystallisation nucleus.

The second, more pronounced, divergence from the theory for $\rho = 0.817$ is caused by the effect of turbulence of cold fluid flowing concentric on the complex surface. This turbulence, in the form of the inlet vortex ring, was indicated by the round recess between the round corner of the plate and the hemisphere. In this surface configuration the entry spin of the fluid intensifies heat transfer on the whole hemisphere. This explanation is confirmed by the exponent equal 1/3 which better approximate the experimental results, expressed in the form of Nusselt–Rayleigh relation, than the exponent of value 1/4, which is correct for the laminar range. Similar intensification effect of convective heat transfer was observed by the authors for a horizontal round heating plate screened by the cylindrical, vertical wall of the height of value $H/D = 0.09$ [8, 9].

CONCLUSION

The proposed model of the phenomenon of convective heat transfer from the complex surface and the solution of this model exhibit a convergence with the results of experimental investigations of horizontal ring with hemisphere in the middle as an example of complex surface. This fact of agreement of theoretical and experimental results may be the proof of verification of presented method. This method allows an estimation of the heat transfer coefficient from any complex surfaces, not only from presented in this paper (example surfaces in Fig. 1).

The augmentation of heat transfer from horizontal ring with hemisphere in the middle with shape factor $\rho = 0.183$ and $\rho = 0.817$ found out in this study may take full advantage in electronics. The intensification for the $\rho = 0.183$ is about 19% in comparison with the round plate and for the $\rho = 0.817$ about 70% compared with a hemisphere. This effect can be used for example for cooling by natural convection of e.g. transistors, diodes, integrated circuits or the electronic devices.

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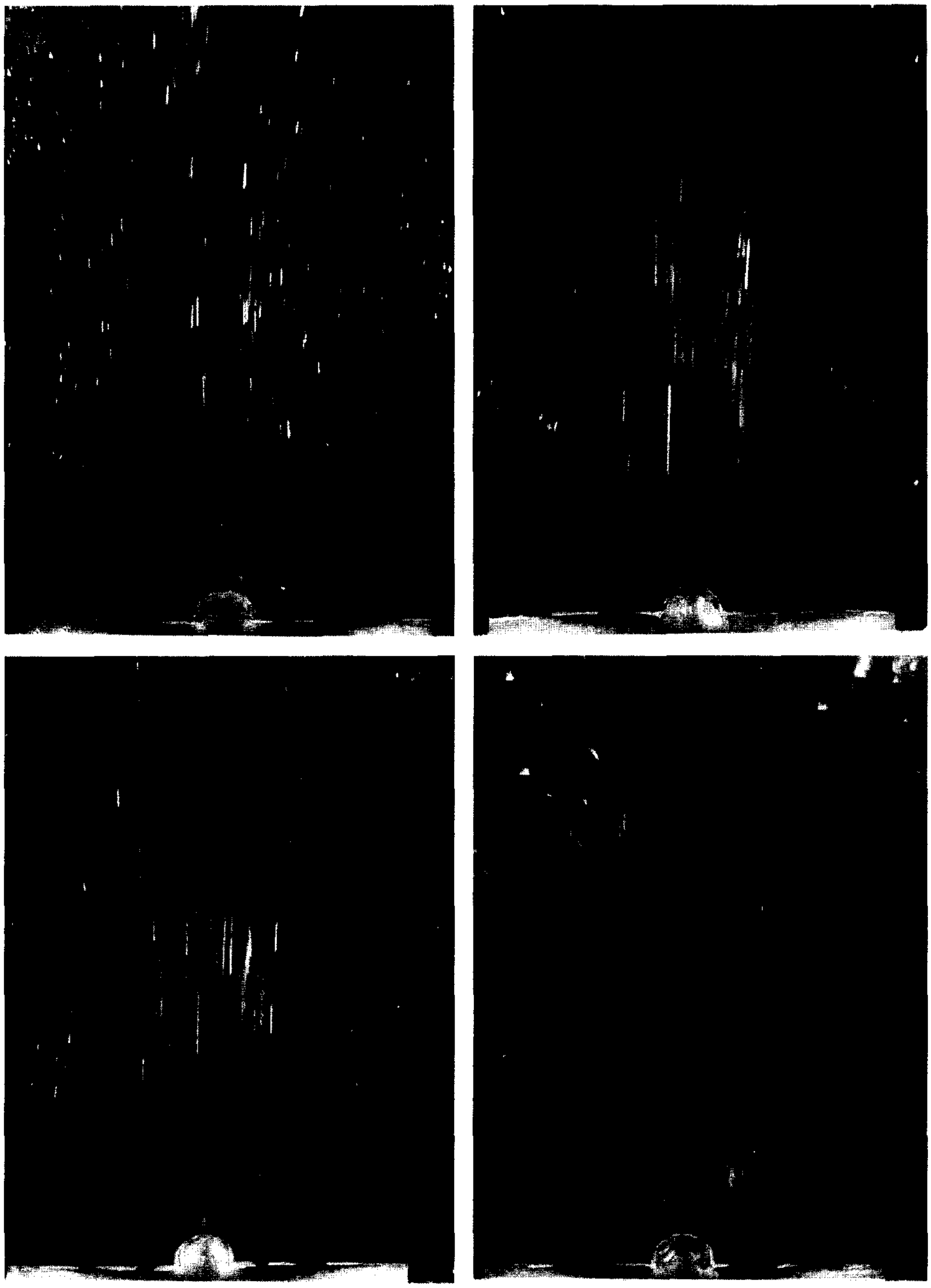
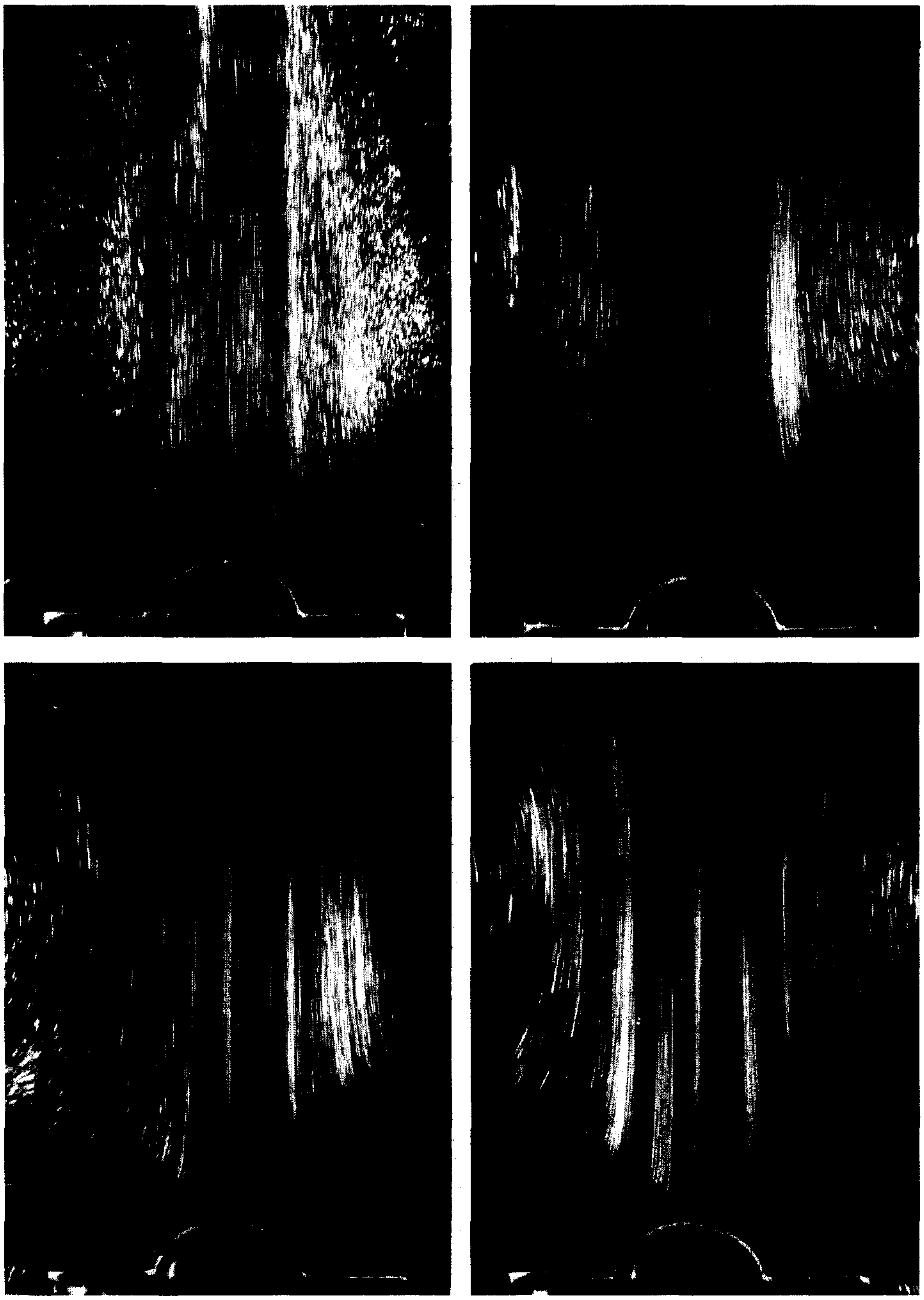


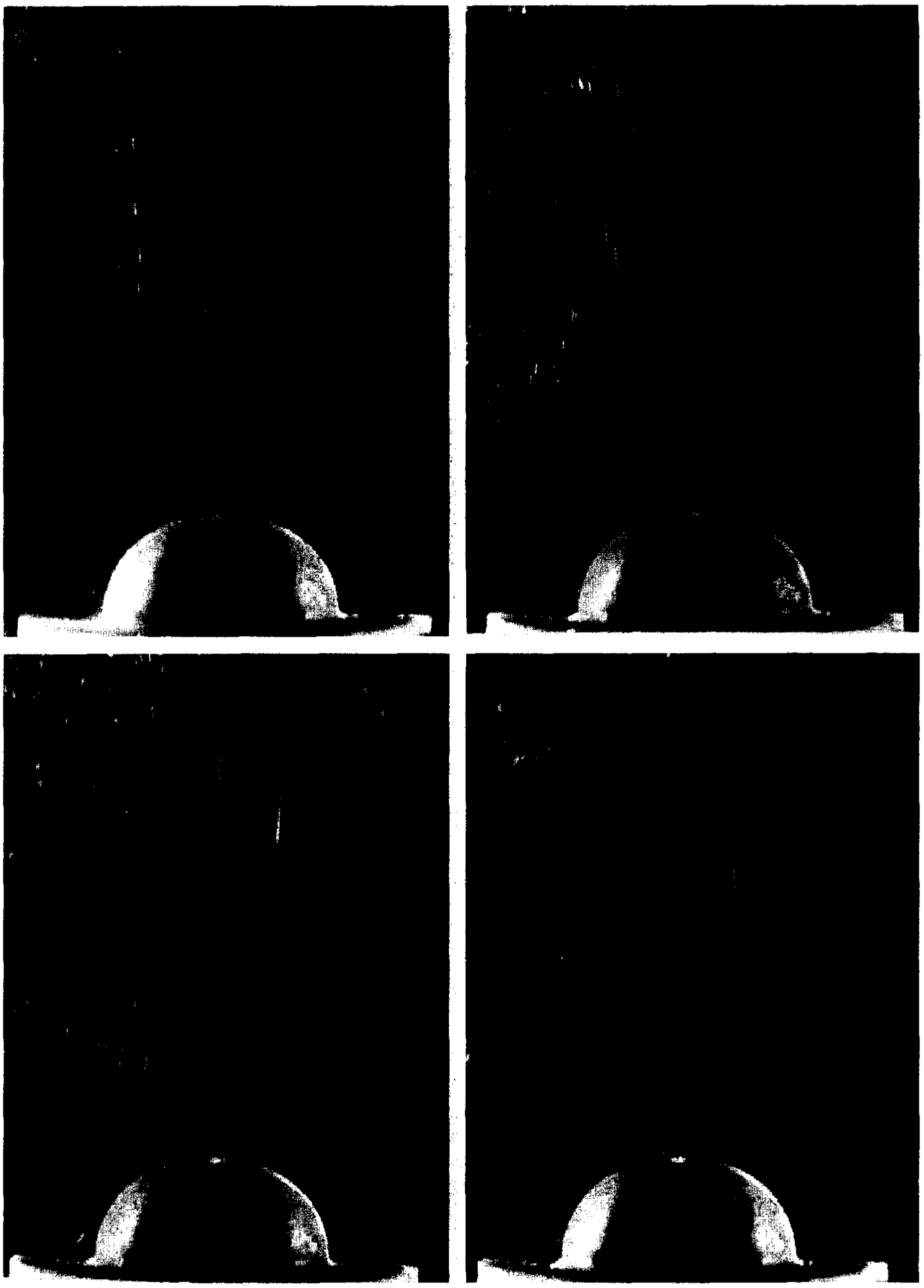
$\rho = 0.183$

Fig. 9. Photographs of some experimental points according to notation by the letters in Fig. 7. The tested fluid was glycerine. Flow pattern marked by aluminium powder and illuminated by laser light–slit was detected by photo-camera. The exposure time was $\tau = 8$ s.



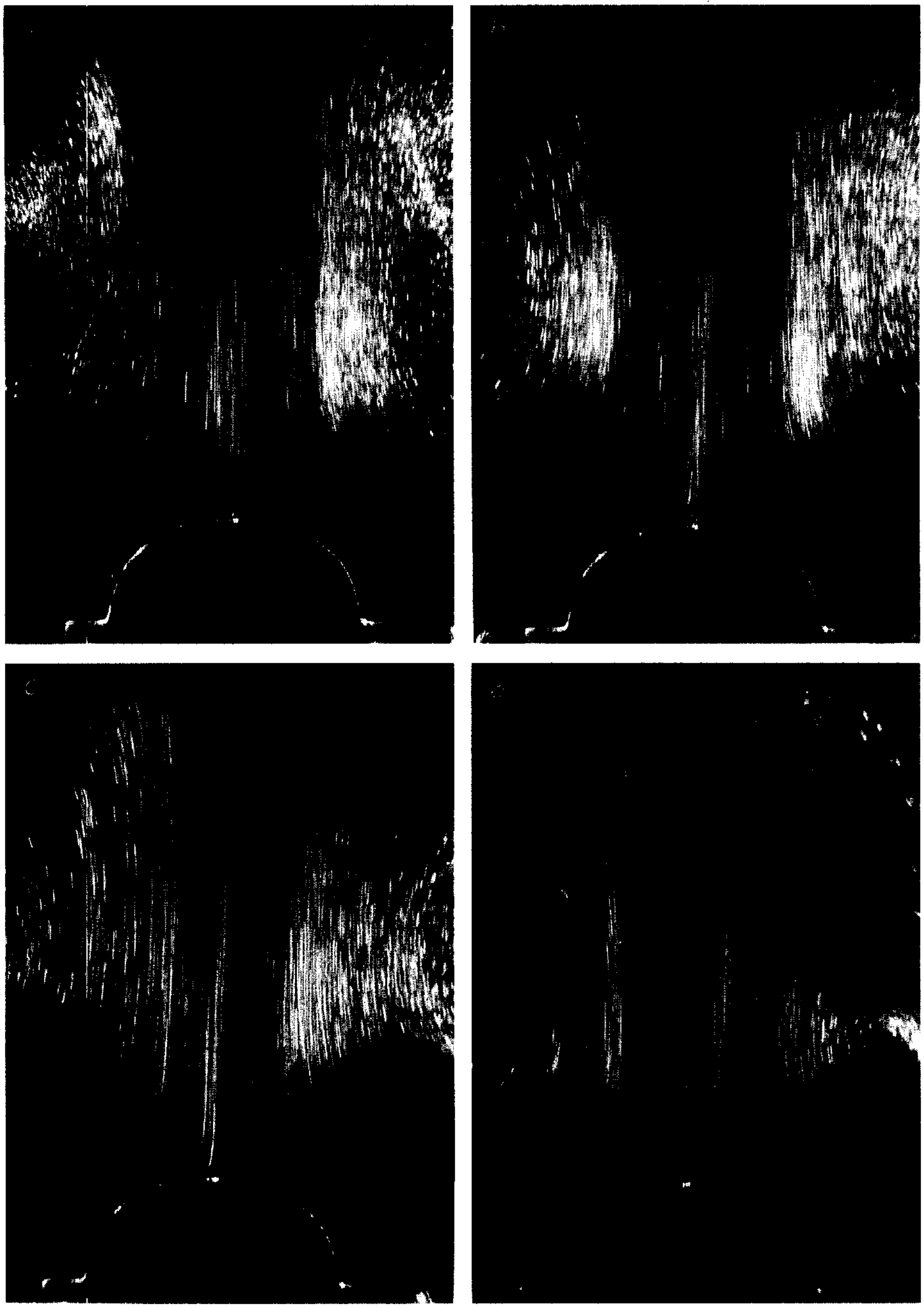
$\rho = 0.483$

Fig. 9—*continued.*



$\rho = 0.65$

Fig. 9—*continued*.



$\rho = 0.817$
Fig. 9—*continued.*

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